Position function

A common application of derivatives is the relationship between speed, velocity and acceleration. In these problems, you're usually given a position equation in the form "$x =$ " or "$s(t) =$ ", which tells you the object's distance from some reference point. This equation also accounts for direction, so the distance could be negative, depending on which direction your object moved away from the reference point.

Average speed of the object is

$$\text{average speed} = \frac{\text{distance}}{\text{time}}$$

Average velocity of the object is

$$\frac{\text{change in position}}{\text{change in time}} = \frac{s(b) - s(a)}{b - a}$$

To find velocity, take the derivative of your original position equation. Speed is the absolute value of velocity. Velocity accounts for the direction of movement, so it can be negative. It's like speed, but in a particular direction. Speed, on the other hand, can never be negative because it doesn't account for direction, which is why speed is the absolute value of velocity. To find acceleration, take the derivative of velocity.

Example

Suppose a particle is moving along the x-axis so that its position at time t is given by the formula

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$$s(t) = 3t^2 + 8t - 2t^{\frac{5}{2}}$$

Compute its velocity and acceleration as functions of t. Next, decide in which direction (left or right) the particle is moving when $t = 1$ and whether its velocity and speed are increasing or decreasing.

To find velocity, we take the derivative of the original position equation.

$$v(t) = s'(t) = 6t + 8 - 5t^{\frac{3}{2}}$$

To find acceleration, we take the derivative of the velocity function.

$$a(t) = v'(t) = s''(t) = 6 - \frac{15}{2}t^{\frac{1}{2}}$$

To determine the direction of the particle at $t = 1$, we plug 1 into the velocity function.

$$v(1) = 6(1) + 8 - 5(1)^{\frac{3}{2}}$$

$$v(1) = 9$$

Because $v(1)$ is positive, we can conclude that the particle is moving in the positive direction (toward the right).

To determine whether velocity is increasing or decreasing, we plug 1 into the acceleration function, because that will give us the rate of change of velocity, since acceleration is the derivative of velocity.

$$a(1) = 6 - \frac{15}{2}(1)^{\frac{1}{2}}$$

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$$a(1) = -\frac{3}{2}$$

Since acceleration is negative at $t = 1$, velocity must be decreasing at that point.

Since the velocity is positive and decreasing at $t = 1$, that means that speed is also decreasing at that point.

Instantaneous velocity

We use the term "instantaneous velocity" to describe the velocity of an object at a particular instant in time. Given an equation that models an object's position over time, $s(t)$, we can take its derivative to get velocity, $s'(t) = v(t)$. We can then plug in a specific value for time to calculate instantaneous velocity.

Example

Given an object's position equation,

$$s(t) = 5t^3 + 2t - 8$$

where s is measured in meters and t is measured in seconds,

1. What is the object's velocity when $t = 0$?
2. What is the object's velocity when $t = 25$?
3. If the motion begins at 8:45 a.m., what is the velocity at 8:51 a.m.?

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First we need to find the derivative of $s(t)$.

$$s'(t) = v(t) = 15t^2 + 2$$

To find velocity when $t = 0$, we'll plug $t = 0$ into $v(t)$.

$$v(0) = 15(0)^2 + 2$$

$$v(0) = 2$$

Velocity at $t = 0$ is 2 m/s.

To find velocity when $t = 25$, we'll plug $t = 25$ into $v(t)$.

$$v(25) = 15(25)^2 + 2$$

$$v(25) = 9{,}377$$

Velocity at $t = 25$ is $9{,}377$ m/s.

8:45 a.m. is when motion begins, which means that time corresponds with $t = 0$. The question asks for velocity after 6 minutes have passed, at 8:51 a.m., but in our position equation, t is measured in seconds. We'll convert minutes into seconds.

$$t = (6 \text{ min})\left(\frac{60\ s}{1 \text{ min}}\right)$$

$$t = 360 \text{ s}$$

Now we can plug $t = 360$ into $v(t)$.

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$$v(360) = 15(360)^2 + 2$$

$$v(360) = 1{,}944{,}002$$

The velocity at 8:51 a.m. is $1{,}944{,}002$ m/s.

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